Class Problem 15.1

- Assume that consumers are identical in all respects expect for their loss probabilities; some are high risk, and others are low risk.
  - Members of the high-risk group have loss probability p_H = 65%, whereas members of the low risk group have loss probability p_L = 35%.
- Each consumer has initial wealth of \$100 and utility $U(W)=W^{0.5}$.
- There are only two possible states of the world, loss and no loss. If a loss occurs, then consumers lose their initial wealth of \$100.
- Insurance contract offerings
  - Policy A provides full coverage for a price of \$65.
  - Policy B provides full coverage for a price of \$45.50.
  - Policy C provides 60% coverage for a price of \$39.
  - Policy D provides 30% coverage for a price of \$13.65.
- Which policy pair should you offer, assuming that you are interested in maximizing profit?

SOLUTION: To answer this question, we must compute the expected utility associated with Policies A through D for high and low risk consumers:

High Risk		**Remain Uninsured**	**Policy A**	**Policy B**	**Policy C**	**Policy D**
p_s	L_s	$U(W_s)$	$U(W_s)$	$U(W_s)$	$U(W_s)$	$U(W_s)$
65%	\$100	$U(0) = 0$	$U(35) = 5.92$	$U(54.50) = 7.38$	$U(21) = 4.58$	$U(16.35) = 4.04$
35%	\$0	$U(100) = 10$	$U(35) = 5.92$	$U(54.50) = 7.38$	$U(61) = 7.81$	$U(86.35) = 9.29$
	$E(U(W))$	**3.50**	**5.92**	**7.38**	**5.71**	**5.88**

Low Risk		**Remain Uninsured**	**Policy A**	**Policy B**	**Policy C**	**Policy D**
p_s	L_s	$U(W_s)$	$U(W_s)$	$U(W_s)$	$U(W_s)$	$U(W_s)$
35%	\$100	$U(0) = 0$	$U(35) = 5.92$	$U(54.50) = 7.38$	$U(21) = 4.58$	$U(16.35) = 4.04$
65%	\$0	$U(100) = 10$	$U(35) = 5.92$	$U(54.50) = 7.38$	$U(61) = 7.81$	$U(86.35) = 9.29$
	$E(U(W))$	**6.50**	**5.92**	**7.38**	**6.68**	**7.46**

Let me explain where the state-contingent wealth values (W_s) come from under the four alternatives (remain uninsured, policy A, policy B, policy C, and policy D). Under policies A and B, there is full insurance coverage, so $W_s = W_0 - P_A$ = \$100 - \$65 = \$35 for both states if policy A is purchased, and $W_s = W_0 - P_B$ = \$100 - \$45.50 = \$54.40 for both states if policy B is purchased. If policy C is purchased, then $W_s = W_0 - P_C - .4(L)$ = \$100 - \$39 - \$40 = \$21 if a loss occurs and $W_s = W_0 - P_C$ = \$100 - \$39 = \$61 otherwise. Finally, if policy D is purchased, then $W_s = W_0 - P_D - .7(L)$ = \$100 - \$13.65 - \$70 = \$16.35 if a loss occurs and $W_s = W_0 - P_D$ = \$100 - \$13.65 = \$86.35 otherwise. In the table above, I have calculated the utility values for all possible values of W_s under the various contract choices. Expected utility ($E(U(W))$) is calculated by multiplying state-contingent utilities by their probabilities and adding these products up; i.e., $E(U(W)) = p(U(W_1)) + (1-p)(U(W_2))$.

Since $E(L)$ is \$65 for high risk consumers, policy A is an actuarially fair policy offering full coverage for such risks. Policy B represents an actuarially unfair policy for low risk consumers, providing full coverage for \$45.50, which represents a 30 percent premium loading over and above the actuarially fair price of \$35. Policy C represents an actuarially fair partial insurance policy for high risk consumers. Finally, policy D is an unfair partial insurance policy for low risk consumers.

You *would not* offer policy B, since high risk consumers will prefer this over policy A and consequently there would be adverse selection. We know from the Bernoulli principle that since policy A is actuarially fair for high risk consumers, they will want to purchase this contract. However, if given the choice between full coverage at \$65 (which high risk consumers are quite happy with) and a full coverage contract for only \$45.50, obviously they would opt for policy B instead of policy A if it were available. However, it won't be available because the insurer can expect to lose \$19.50 on each policy sold to high risk consumers.

Either policy C or D could be offered, since the high risk consumers will prefer policy A to the alternatives of remaining uninsured or buying either of these contracts, since the expected utility is highest for policy A. Low risk consumers will prefer to partially insure using either policy C or policy D, since the expected utilities for both of these contracts exceed the expected utilities of remaining uninsured or buying policy A. However, you would make the most profit by offering policy C, since the expected cost of the loss on every contract sold to low risk consumers is $.60p_L L = .60(.35)\$100 = \$21$, and you receive a premium payment of \$39. Thus the expected profit from offering policy C is \$39-\$21 = \$18. Low risk consumers would prefer that you offer policy D instead of policy C, since it provides higher expected utility. However, since the expected cost of the loss on every D contract sold to low risk consumers is $.30p_L L = .30(.35)\$100 = \$10.50$, you would expect to earn only \$13.65 - \$10.50 = \$3.15 in profit per low risk consumer. Therefore, you'll want to offer contracts A and C. This way, there is no adverse selection, and although you'll break even on the high risk consumers, you'll make the maximum possible profit on the low risk consumers.